4. [12 points] During a party, the host discovers that he has been robbed of his favorite gold hallway clock, and immediately calls the police. The last time the host noticed the clock was still hanging, it was 6:00 p.m. When the police arrive, Officer Tom notices a decorative ice sculpture. When he first arrives at 9:00 p.m., the ice sculpture's temperature is 27°F. After questioning others at the party, Officer Tom again takes the temperature of the ice sculpture at 10:00 p.m., and finds it to be 29°F. The temperature of the room has remained a constant 68°F all day.

a. [2 points] Let S be the temperature of the sculpture, measured in degrees Fahrenheit. Assuming the sculpture obeys Newton's Law of Heating and Cooling, write a differential for $\frac{dS}{dt}$, where t is the number of hours since 9:00 p.m. Your answer may contain an unknown constant, k.

Solution: $\frac{dS}{dt} = k(S - 68)$ (or some appropriate variation thereof)

b. [7 points] Using separation of variables, and the information provided about the sculpture, solve the differential equation to find $S(t)$, where t is the number of hours since 9:00 p.m. Your answer should contain no unknown constants.

Solution: $\frac{dS}{S-68} = kdt$, which gives $\ln|S - 68| = kt + C$, or $S = Ae^{kt} + 68$. When $t = 0$, $S = 27$, which gives us $27 = A + 68$, so $A = -41$, leaving $S = -41e^{kt} + 68$. We use the other condition to solve for k. At $t = 1$, $S = 29$, so $29 = -41e^{k(1)} + 68$. We solve for k: $41e^k = 39$, or $e^k = \frac{39}{41}$, which leaves us with $k = \ln\left(\frac{39}{41}\right) \approx -0.05001$.

$$S = -41e^{-0.05001t} + 68$$

c. [3 points] Company FunIce provided the sculpture, and it was delivered by their employee Bill. For each ice sculpture they produce, the company guarantees the temperature will be exactly 18°F upon delivery. If the sculpture was 18°F at delivery, and Bill left directly afterwards, should he be considered as a possible suspect of the robbery? Briefly justify your answer.

Solution: We solve for when the temperature of the sculpture was $S = 18$. We solve for t: $18 = -41e^{-0.05001t} + 68$, or $\frac{50}{41} = e^{-0.05001t}$. This gives us $t \approx -3.96823$ hours. The sculpture was delivered nearly four hours before 9:00 p.m., putting the delivery near 5:00 p.m. Since Bill left immediately after delivery, he should not be considered as a suspect, since the clock was still there at 6:00 p.m.