6. [12 points] A certain sculpture is the volume of revolution of the function $y = f(x)$ about the $x-$axis. The function $f(x)$ is twice differentiable, increasing, concave-up, and has the values given below, where all measurements are in meters:

x	0	0.25	0.5	0.75	1
$f(x)$	0.2	0.25	0.3	0.4	0.6

a. [3 points] Suppose the density of the sculpture is 2400 kg/m^3. Write an integral expression for the total mass of the sculpture.

Solution: $\int_0^1 2400\pi f(x)^2 dx$

b. [3 points] Approximate the total mass of the sculpture using the *midpoint* rule with as many subdivisions as possible, given the data. Include appropriate units.

Solution: The most accurate use of the midpoint rule has only two subintervals. So

$$\begin{aligned}\int_0^1 2400\pi f(x)^2 dx &\approx 2400\pi \cdot 0.5 \cdot (f(0.25)^2 + f(0.75)^2) \\ &= 2400\pi \cdot 0.5 \cdot (0.625 + 0.16) \\ &\approx 838.805 \text{ kg}\end{aligned}$$

c. [3 points] Approximate the moment of the sculpture about its axis of symmetry using the *trapezoid* rule with as many subdivisions as possible, given the data.

Solution:

$$\begin{aligned}\int_0^1 2400\pi x f(x)^2 dx &\approx 2400\pi \cdot 0.25 \cdot (\frac{1}{2}0f(0)^2 + 0.25f(0.25)^2 + 0.5f(0.5)^2 \\ &\quad +0.75f(0.75)^2 + \frac{1}{2}1f(1)^2) \\ &\approx 679.762\end{aligned}$$

d. [3 points] Once completed, the sculpture will be arranged so that the end corresponding to $x = 0$ will rest on the ground. If the center of mass is more than half of the height of the sculpture, then the sculptor will have to brace the sculpture with extra supports. Approximate how high (in meters) the center of mass will be above the ground once the sculpture is placed on its $x = 0$ end using your answers from the previous two parts. Will the sculptor need to provide extra support?

Solution:

$$\overline{x} = 679.762/838.805 \approx 0.8104$$

The center of mass is approximately 0.8104 meters above the ground, which is more than 0.5 meters, half the height of the sculpture. The sculptor will need to provide extra support.