1. [11 points] Indicate if each of the following is true or false by circling the correct answer. Justify your answer.

a. [2 points] If the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n (x - 4)^n$ is 2, then $\sum_{n=0}^{\infty} a_n$ diverges.

True  False

Solution: $x = 5$ belongs to the interval of convergence then $\sum_{n=0}^{\infty} a_n$ converges.

b. [2 points] If $P(x)$ is a cumulative distribution function with $P(0) = \frac{1}{3}$, then the median is positive.

True  False

Solution: Since $P(x)$ is increasing and $P(\text{median}) = \frac{1}{2}$ then median > 0.

c. [3 points] If $F(x) = \int_{-x^2}^{0} \frac{1}{1+t^4} dt$ then $F(x)$ is decreasing for $x > 0$.

True  False

Solution: $F'(x) = \frac{2x}{1+x^8} > 0$ for $x > 0$. Hence $F(x)$ is increasing for $x > 0$.

d. [2 points] The differential equation $y' = (y - x^3)y$ has two equilibrium solutions, $y = 0$ and $y = x^3$.

True  False

Solution: $y = x^3$ is not an equilibrium solution.

e. [2 points] Using the slope field below, we can guarantee that the solution with initial condition $y(0) = \frac{1}{2}$ satisfies $y(3) < 0$.

True  False

Solution: $y' > 0$ for $y < 0$, hence $y(3) \geq 0$. 