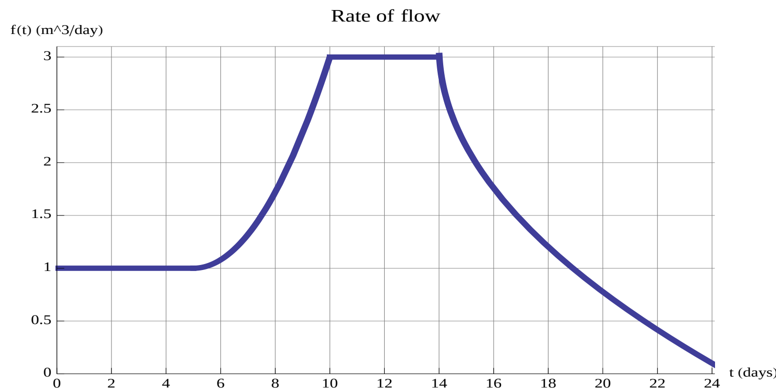
3. [11 points] Sewage flows into the tank described in the previous problem at a rate of $f(t)$ cubic meters per day. Let t be the number of days since December 1, when the tank had 1 m^3 of sewage. A graph of $f(t)$ is given below. Use it to answer the following questions.



a. [3 points] Suppose that $V(t)$ gives the volume of sewage in the tank at time t. Find a formula for $V(t)$ in terms of $f(t)$.

Solution: $V(t) = 1 + \int_0^t f(t)dt$

b. [2 points] For what times t in $[0, 24]$ is $V(t)$ concave up? ____________________

Solution: $4 < t < 10$

c. [2 points] For what times t in $[0, 24]$ is $V(t)$ concave down? ____________________

Solution: $14 < t < 24$

d. [4 points] Fill out the table below. Using the values in your table, compute Riemann sums with 3 subintervals to find an underestimate and an overestimate for $V(12)$. Justify why the Riemann sums you selected yield the appropriate under and upper estimates. Do not forget to include the units in your answer.

t	0	4	8	12
$f(t)$				

Solution:

t	0	4	8	12
$f(t)$	1	1	$\approx$ 1.75	3

$f(t)$ is increasing in $(0, 12)$ then
Upper estimate (Right hand sum): $4(1 + 1.75 + 3) + 1 = 24\ m^3$
Lower estimate (Left hand sum): $4(1 + 1 + 1.75) + 1 = 16\ m^3$