5. [9 points]

a. [3 points] In the picture below, the graphs of \( r = 2 \) and \( r = 2 - \sin(5\theta) \) are shown. Write a definite integral that computes the shaded area.

Solution: Endpoints satisfy: \( 2 = 2 - \sin(5\theta) \), hence \( \sin(5\theta) = 0 \). Then the endpoints are \( \theta = \frac{\pi}{5}, \frac{2\pi}{5} = 0.628, 1.25 \) radians.

\[
A = \frac{1}{2} \int_{\frac{\pi}{5}}^{\frac{2\pi}{5}} (2 - \sin(5\theta))^2 - 4d\theta
\]

b. [6 points] Find parametric equations of the tangent line to the limacon \( r = \sin(\theta) - \frac{1}{2} \) at \( \theta = \frac{\pi}{4} \).

Solution:

\[
x(\theta) = (\sin \theta - \frac{1}{2}) \cos \theta \\
x'(\theta) = (\sin \theta - \frac{1}{2})(-\sin \theta) + \cos^2(\theta) \\
x'(\frac{\pi}{4}) = .353 \\
y(\theta) = (\sin \theta - \frac{1}{2}) \sin \theta \\
y'(\theta) = (\sin \theta - \frac{1}{2}) \cos \theta + \sin \theta \cos \theta \\
y'(\frac{\pi}{4}) = .646 \\
x_{\tan}(t) = .146 + .353 t \\
y_{\tan}(t) = .146 + .646 t
\]