7. [14 points] For each of the following sequences

1.Compute $\lim_{n\to\infty} a_n$.

2.Decide if $\sum_{n=0}^{\infty} a_n$ converges or diverges. Circle your answer.

Support your answer by stating the test(s) or facts you used to prove convergence or divergence , and show complete work and justification.

a. [4 points]

$$a_n = \left(\frac{-1}{\pi}\right)^n \qquad \lim_{n\to\infty} a_n = \_\_\_\_\_\_ \qquad \sum_{n=0}^{\infty} a_n : \quad \text{Converges} \quad \text{Diverges}$$

Solution: Sequence: $a_n = r^n$ where $|r| = \left|\frac{-1}{\pi}\right| = .318 < 1$ hence $\lim_{n\to\infty} a_n = 0$.

Series: $\sum_{n=0}^{\infty} a_n = \sum_{n=0}^{\infty} r^n$ is a geometric series with $|r| < 1$ then it **converges**.

b. [4 points]

$$a_n = \frac{n^2+2}{1+4n^2} \qquad \lim_{n\to\infty} a_n = \_\_\_\_\_\_ \qquad \sum_{n=0}^{\infty} a_n : \quad \text{Converges} \quad \text{Diverges}$$

Solution: Sequence: $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n^2+2}{1+4n^2} = \lim_{n\to\infty} \frac{n^2}{4n^2} = \frac{1}{4}$.

Series: Since a_n does not converge to 0 then $\sum_{n=0}^{\infty} a_n$ diverges.

Note to the graders: The criteria for the justification of the divergence of the series has been given different names in some sections (nth term test and some others). If you see these kind of justifications, please ask the instructor before considering any deductions.

c. [6 points]

$$a_n = \frac{n}{\sqrt{n^4+5}} \qquad \lim_{n\to\infty} a_n = \_\_\_\_\_\_ \qquad \sum_{n=0}^{\infty} a_n : \quad \text{Converges} \quad \text{Diverges}$$

Solution: Sequence: $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{\sqrt{n^4+5}} = \lim_{n\to\infty} \frac{n}{n^2} = 0$.

Series: $a_n = \frac{n}{\sqrt{n^4+5}} \sim \frac{1}{n}$ as n goes to infinity.
Limit Comparison Test:

$$\lim_{n\to\infty} \frac{a_n}{\frac{1}{n}} = \lim_{n\to\infty} \frac{\frac{n}{\sqrt{n^4+5}}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^2}{\sqrt{n^4+5}} = \lim_{n\to\infty} \frac{n^2}{n^2} = 1.$$

$\sum_{n=0}^{\infty} a_n \sim \sum_{n=0}^{\infty} \frac{1}{n}$. Hence diverges.