5. [10 points] When a voltage V in volts is applied to a series circuit consisting of a resistor with resistance R in ohms and an inductor with inductance L, the current I(t) at time t is given by

$$I(t) = \frac{V}{R} \left(1 - e^{-\frac{Rt}{L}} \right) \qquad \text{ where } V, R, \text{ and } L \text{ are constants.}$$

a. [2 points] Show that I(t) satisfies

$$\frac{dI}{dt} = \frac{V}{L} \left(1 - \frac{R}{V}I \right).$$

b. [6 points] Find a Taylor series for I(t) about t=0. Write the first three nonzero terms and a general term of the Taylor series.

c. [2 points] Use the Taylor series to compute

$$\lim_{t \to 0} \frac{I(t)}{t}.$$