10. [7 points] A metal thin plate has density \( \delta(x) = 1 + x \) kg per square meter. The shape of the plate is bounded by the curves \( y = 1 + x \) and \( y = 1 + x^2 \) for \( 0 \leq x \leq 1 \) as shown below.

\[
\begin{align*}
\Delta x \Delta y &= (1 + x - 1 - x^2)(1 + x) \Delta x \\
&= (x - x^2)(1 + x) \Delta x \\
&= x - x^3 \Delta x \\
&= x^4 - x^6 \\
&= \left[ \frac{1}{2} - \frac{1}{4} \right] = \frac{1}{4} \text{ kg}.
\end{align*}
\]

b. [4 points] Let \((\bar{x}, \bar{y})\) be its center of mass. Write a formula for \(\bar{x}\) and evaluate your formula to find the exact value of \(\bar{x}\). Show all your work.

\[
\bar{x} = \frac{\int_0^1 x(x - x^3) dx}{\int_0^1 (x - x^3) dx} = 4 \int_0^1 x^2 - x^4 dx = 4 \left( \frac{x^3}{3} - \frac{x^5}{5} \right) \bigg|_0^1 = 4 \left( \frac{1}{3} - \frac{1}{5} \right) = \frac{8}{15} \text{ meters}.
\]