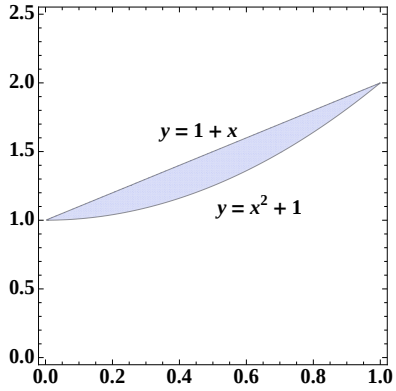
10. [7 points] A metal thin plate has density $\delta(x) = 1 + x$ kg per square meter. The shape of the plate is bounded by the curves $y = 1 + x$ and $y = 1 + x^2$ for $0 \leq x \leq 1$ as shown below.



a. [3 points] Find the exact value of the mass of the plate. Show all your work.

Solution:

$$m = \int_0^1 (1 + x - 1 - x^2)(1 + x)dx = \int_0^1 (x - x^2)(1 + x)dx = \int_0^1 x - x^3 dx$$
$$= \frac{x}{2} - \frac{x^4}{4}\Big|_0^1 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4} \text{ kg.}$$

b. [4 points] Let $(\bar{x}, \bar{y})$ be its center of mass. Write a formula for $\bar{x}$ and evaluate your formula to find the exact value of $\overline{x}$. Show all your work.

Solution:

$$\bar{x} = \frac{\int_0^1 x(x - x^3)dx}{\frac{1}{4}} = 4\int_0^1 x^2 - x^4 dx = 4\left(\frac{x^3}{3} - \frac{x^5}{5}\right)\Big|_0^1 = 4\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{8}{15} \text{ meters .}$$