4. [9 points] Ramon starts depositing \$10,000 each year at his 25th birthday into a retirement account and continues until his 45th birthday. After this point, he does not touch the account until he is 65. The retirement account accrues interest at a rate of 3% compounded annually.

a. [3 points] Let R_n be the amount of money *in thousands* of dollars in Ramon's retirement account after n years from his initial deposit. Find an expression for R_0, R_1 and R_2.

Solution:
$R_0 = 10$, since Ramon deposits \$10,000 initially.
$R_1 = 10 + 10(1.03)$, since Ramon deposits another \$10,000 and the previous year's deposit accrues interest.
$R_2 = 10 + [10 + 10(1.03)]\,(1.03) = 10 + 10(1.03) + 10(1.03)^2$, since all of R_1 accrues interest.

b. [3 points] Find a closed form expression (an expression that does not involve a long summation) for how much money Ramon has in his retirement account at his 45th birthday.

Solution: From the calculations in part (a), we can see that

$$R_n = 10 + 10(1.03) + 10(1.03)^2 + \ldots + 10(1.03)^n.$$

Ramon's 45th birthday corresponds to $n = 20$, so

$$R_{20} = 10 + 10(1.03) + 10(1.03)^2 + \ldots + 10(1.03)^{20} = \sum_{k=0}^{20} 10(1.03)^k.$$

As this is a finite geometric series with 21 terms, a closed form expression is

$$R_{20} = \frac{10(1 - 1.03^{21})}{1 - 1.03},$$

where the amount is given in thousands of dollars.

c. [3 points] Find a closed form expression for how much money Ramon has in his retirement account when he is 65 years old. Compute its value.

Solution: Since Ramon stops depositing money after his 45th birthday, his account is just accumulating interest (at an annual rate of 3%) for the next 20 years. Thus, when he is 65 years old, his account balance is

$$R_{20}\,(1.03)^{20} = \frac{10(1 - 1.03^{21})}{1 - 1.03}\,(1.03)^{20} \approx 517.92923$$

in thousands of dollars (\$517,929.23).