6. [7 points] Let \( f(x) = \frac{1}{3x-2} \) and \( g(x) \) be the function whose graph is shown below.

![Graph](image)

a. [3 points] Let \( P_2(x) = a + b(x - 2) + c(x - 2)^2 \) be the second order Taylor polynomial approximating \( g(x) \) for \( x \) near 2. What can you say about the signs of the coefficients \( a \), \( b \) and \( c \)?

Solution: Since \( P_2(x) = g(2) + g'(2)(x - 2) + \frac{g''(2)}{2}(x - 2)^2 \), then \( a = g(2) \), \( b = g'(2) \) and \( c = \frac{g''(2)}{2} \). At \( x = 2 \) the function \( g(x) \) is positive \( (g(2) > 0) \), (increasing \( g'(2) > 0 \)) and concave down \( (g''(2) < 0) \). Hence \( a > 0 \), \( b > 0 \) and \( c < 0 \).

b. [4 points] Find the second order Taylor polynomial approximating \( f(x) \) for \( x \) near \(-1\).

Solution:

\[
\begin{align*}
  f(x) &= \frac{1}{3x-2} \quad f(-1) = -\frac{1}{5} \\
  f'(x) &= -\frac{3}{(3x-2)^2} \quad f'(-1) = -\frac{3}{25} \\
  f''(x) &= \frac{18}{(3x-2)^3} \quad f''(-1) = -\frac{18}{125}.
\end{align*}
\]

Hence \( P_2(x) = f(-1) + f'(-1)(x + 1) + \frac{f''(-1)}{2}(x + 1)^2 \)

\[
P_2(x) = -\frac{1}{5} - \frac{3}{25}(x + 1) - \frac{9}{125}(x + 1)^2.
\]