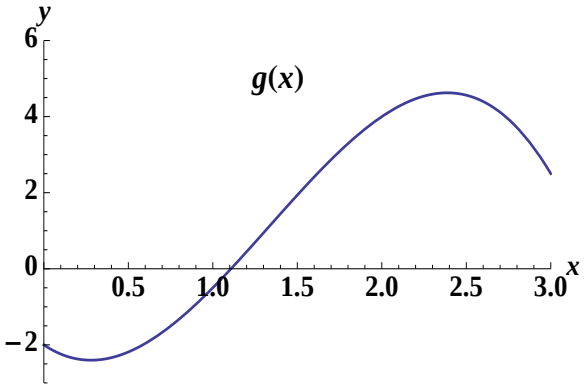
6. [7 points] Let $f(x) = \frac{1}{3x-2}$ and $g(x)$ be the function whose graph is shown below.



a. [3 points] Let $P_2(x) = a + b(x-2) + c(x-2)^2$ be the second order Taylor polynomial approximating $g(x)$ for x near 2. What can you say about the signs of the coefficients a, b and c?

Solution: Since $P_2(x) = g(2) + g'(2)(x-2) + \frac{g''(2)}{2}(x-2)^2$, then $a = g(2)$, $b = g'(2)$ and $c = \frac{g''(2)}{2}$. At $x = 2$ the function $g(x)$ is positive ($g(2) > 0$), (increasing $g'(2) > 0$) and concave down ($g''(2) < 0$). Hence $a > 0$, $b > 0$ and $c < 0$.

b. [4 points] Find the second order Taylor polynomial approximating $f(x)$ for x near -1.

Solution:

$$f(x) = \frac{1}{3x-2} \qquad f(-1) = -\frac{1}{5}$$
$$f'(x) = \frac{-3}{(3x-2)^2} \qquad f'(-1) = -\frac{3}{25}$$
$$f''(x) = \frac{18}{(3x-2)^3} \qquad f''(-1) = -\frac{18}{125}.$$

Hence $P_2(x) = f(-1) + f'(-1)(x+1) + \frac{f''(-1)}{2}(x+1)^2$

$$P_2(x) = -\frac{1}{5} - \frac{3}{25}(x+1) - \frac{9}{125}(x+1)^2.$$