

9. [11 points] Philip J. Frye has a bank account with Big Apple Bank that compounds interest at a continuous annual rate of 1%. His account has a balance of \$300 at midnight of January 1, 2000, when Frye is cryogenically frozen for 1000 years. The entire time he is frozen, his account accumulates interest. Include units in your answers where appropriate.

- a. [2 points] Write a differential equation that models Frye's account balance  $M(t)$ , where  $M$  is given in dollars and  $t$  is the number of years since January 1, 2000. List any initial conditions.

*Solution:*

$$\frac{dM}{dt} = 0.01M \quad M(0) = 300.$$

- b. [4 points] Solve your differential equation from (a) to find the balance in Frye's account after he is awoken in the year 3000. Show all your work.

*Solution:*

$$\begin{aligned} \frac{dM}{dt} &= 0.01M & \frac{dM}{M} &= 0.01dt & \ln |M| &= 0.01t + C \\ M(t) &= Be^{0.01t} \\ M(0) = 300 & \quad 300 = B & M(t) &= 300e^{0.01t}. \end{aligned}$$

- c. [2 points] Suppose that Big Apple Bank charges an annual fee of \$5 to maintain the account, withdrawn continuously over the course of the year. Write a new differential equation for  $M(t)$ , the balance in Frye's bank account.

*Solution:*

$$\frac{dM}{dt} = 0.01M - 5.$$

- d. [3 points] How large must the initial deposit in Frye's account be at Big Apple Bank in order for the account to be profitable for him? Justify your answer mathematically.

*Solution:* The differential equation has an equilibrium solution at  $M_{eq} = \frac{5}{0.01} = 500$ . The equilibrium solution is unstable since

$$\left. \frac{dM}{dt} \right|_{M=400} = .01(400) - 5 = -1 < 0 \quad \text{and} \quad \left. \frac{dM}{dt} \right|_{M=600} = .01(600) - 5 = 1 > 0$$

Hence the initial deposit  $M(0)$  has to be larger than  $M_{eq} = 500$  dollars in order to be profitable ( $M(0) > 500$ ).