9. [11 points] An object is dropped from a height of 100 meters. If air resistance is considered, the height of the object $y(t)$ (in meters) above the ground t seconds after it was dropped is given by

$$y(t) = 100 - \frac{g}{k}t + \frac{g}{k^2}\left(1 - e^{-kt}\right).$$

where $k > 0$ is a constant representing the intensity of air resistance and $g = 9.8$ m/s^2 is the acceleration due to gravity.

a. [3 points] Show that $y(t)$ satisfies $y'' + ky' + g = 0$.

b. [6 points] Use the first four nonzero terms of the Taylor series of the function $f(t) = e^{-kt}$ about $t = 0$ to find an approximation for $y(t)$.

c. [2 points] Using your answer from part (b), evaluate $\lim_{k\to 0} y(t)$.