

1. [12 points] Indicate if each of the following is true or false by circling the correct answer. No justification is required.

a. [2 points] In polar coordinates, $(r_1, \theta_1) = (2, \frac{\pi}{5})$ and $(r_2, \theta_2) = (-2, -\frac{4\pi}{5})$ represent the same point.

True

False

b. [2 points] If a particle moves according to the parametric equations $x(t) = t^3 + t^2$ and $y(t) = t^4$, then the particle has speed zero at the origin.

True

False

Solution: The speed of the particle is given by $v(t) = \sqrt{(x')^2 + (y')^2}$. In this case $v(t) = \sqrt{(3t^2 + 2t)^2 + (4t^3)^2}$. The particle is at the origin when $t = 0$ and $v(0) = 0$.

c. [2 points] The Taylor series for $f(x) = \sqrt{1 + 2x}$ centered about $x = 0$ converges for $-1 < x < 1$.

True

False

Solution: The Taylor series for $f(x) = \sqrt{1 + 2x}$ centered about $x = 0$ converges for $-\frac{1}{2} < x < \frac{1}{2}$.

d. [2 points] If $P(t)$ is a cumulative distribution function, then the sequence $x_n = P(n)$ converges.

True

False

Solution: If $P(t)$ is a cumulative distribution function, then $\lim_{t \rightarrow \infty} P(t) = 1$, hence $\lim_{n \rightarrow \infty} P(n) = 1$.

e. [2 points] The sequence $a_n = \int_{\frac{1}{n}}^1 \frac{1}{x^3} dx$ converges.

True

False

Solution: $\lim_{t \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \int_{\frac{1}{n}}^1 \frac{1}{x^3} dx = \int_0^1 \frac{1}{x^3} dx$ which diverges by p -series test with $p = 3 > 1$.

f. [2 points] The function $F(x) = \int_1^{x^2} \sin(t^2) dt$ is an even function.

True

False

Solution: $F(-x) = \int_1^{(-x)^2} \sin(t^2) dt = \int_1^{x^2} \sin(t^2) dt = F(x)$.