3. [12 points] Let

$$I = \int_0^1 \left(1 + \frac{t^2}{2}\right)^{\frac{5}{2}} dt$$

a. [5 points] Approximate the value of I using Right(2) and Mid(2). Write each term in your sums.

Solution:

$$\begin{aligned}\text{Right}(2) &= \frac{1}{2}\left(\left(\frac{9}{8}\right)^{\frac{5}{2}} + \left(\frac{3}{2}\right)^{\frac{5}{2}}\right) \\ &\approx \frac{1}{2}(1.342 + 2.755) \approx 2.04904 \\ \text{Mid}(2) &= \frac{1}{2}\left(\left(\frac{33}{32}\right)^{\frac{5}{2}} + \left(\frac{41}{32}\right)^{\frac{5}{2}}\right) \\ &\approx \frac{1}{2}(1.08 + 1.858) \approx 1.46907\end{aligned}$$

b. [2 points] Are your estimates of the value of I obtained using Right(2) and Mid(2) guaranteed to be overestimates, underestimates or neither?

Solution: Right = Overestimate (increasing)
Mid = Underestimate (concave up)

c. [3 points] Find the first three nonzero terms of the Taylor series for $g(t) = \left(1 + \frac{t^2}{2}\right)^{\frac{5}{2}}$ about $t = 0$.

Solution: Using the binomial series:

$$\begin{aligned}(1+x)^{\frac{5}{2}} &= 1 + \frac{5}{2}x + \left(\frac{5}{2}\right)\left(\frac{3}{2}\right)\frac{x^2}{2!} + \cdots \\ &= 1 + \frac{5}{2}x + \frac{15}{8}x^2 + \cdots \\ \left(1 + \frac{t^2}{2}\right)^{\frac{5}{2}} &= 1 + \frac{5}{2}\left(\frac{t^2}{2}\right) + \frac{15}{8}\left(\frac{t^2}{2}\right)^2 + \cdots \\ &= 1 + \frac{5}{4}t^2 + \frac{15}{32}t^4 + \cdots\end{aligned}$$

d. [2 points] Use your answer from part (c) to estimate I.

Solution:

$$I \approx \int_0^1 1 + \frac{5}{4}t^2 + \frac{15}{32}t^4 dt = t + \frac{5}{12}t^3 + \frac{3}{32}t^5\Big|_{t=0}^{1} = 1 + \frac{5}{12} + \frac{3}{32} = \frac{145}{96} \approx 1.510417$$