5. [9 points] Consider the following power series:

$$\sum_{n=1}^{\infty} \frac{1}{n5^n}(x-4)^{n+1}$$

a. [4 points] Find the radius of convergence of the power series. Show all your work.

Solution:

$$\lim_{n\to\infty} \frac{\frac{1}{(n+1)5^{n+1}}}{\frac{1}{n5^n}} \frac{|x-4|^{n+2}}{|x-4|^{n+1}} = \frac{1}{5}|x-4| \lim_{n\to\infty} \frac{n}{n+1} = \frac{1}{5}|x-4|.$$

$$\frac{1}{5}|x-4| < 1 \Leftrightarrow |x-4| < 5,$$

so $R = 5$.

b. [5 points] For which values of x does the series converge absolutely? For which values of x does it converge conditionally?

Solution: Converges absolutely inside radius: $(-1, 9)$.
Left endpoint: $x = -1$,

$$\sum_{n=1}^{\infty} \frac{1}{n5^n}(-5)^{n+1} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n},$$

converges conditionally (alternating harmonic series).
Right endpoint: $x = 9$,

$$\sum_{n=1}^{\infty} \frac{1}{n5^n}5^{n+1} = 5\sum_{n=1}^{\infty} \frac{1}{n},$$

diverges. So, converges conditionally for $x = -1$, absolutely for $-1 < x < 9$.