5. [9 points] Consider the following power series:

\[ \sum_{n=1}^{\infty} \frac{1}{n5^n}(x - 4)^{n+1} \]

a. [4 points] Find the radius of convergence of the power series. Show all your work.

**Solution:**

\[
\lim_{n \to \infty} \frac{\frac{1}{n5^n} |x - 4|^{n+2}}{\frac{1}{(n+1)5^{n+1}} |x - 4|^{n+1}} = \frac{1}{5} |x - 4| \lim_{n \to \infty} \frac{n}{n + 1} = \frac{1}{5} |x - 4|.
\]

\[
\frac{1}{5} |x - 4| < 1 \iff |x - 4| < 5,
\]

so \( R = 5 \).

b. [5 points] For which values of \( x \) does the series converge absolutely? For which values of \( x \) does it converge conditionally?

**Solution:** Converges absolutely inside radius: \((-1, 9)\).

Left endpoint: \( x = -1 \),

\[
\sum_{n=1}^{\infty} \frac{1}{n5^n}(-5)^{n+1} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n},
\]

converges conditionally (alternating harmonic series).

Right endpoint: \( x = 9 \),

\[
\sum_{n=1}^{\infty} \frac{1}{n5^n}5^{n+1} = 5 \sum_{n=1}^{\infty} \frac{1}{n},
\]

diverges. So, converges conditionally for \( x = -1 \), absolutely for \(-1 < x < 9\).