**6**. [12 points] In the following problems, support all you answers by stating the test(s) or facts you used to prove convergence or divergence. Show all your work.

**a**. [4 points] $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{1+n^3}$ Circle your answer: Converges Diverges

*Solution:* Compare to: $\displaystyle\sum_{n=1}^{n} \frac{1}{n^{5/2}}$, converges by $p$-test, $p = \frac{5}{2} > 1$.

$$\lim_{n\to\infty} \frac{\frac{\sqrt{n}}{1+n^3}}{\frac{1}{n^{5/2}}} = \lim_{n\to\infty} \frac{n^3}{1+n^3} = 1 > 0,$$

so series converges by LCT.

**b**. [4 points] $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2+\cos^2(n)}$ Circle your answer: Converges Diverges

*Solution:* $\displaystyle\lim_{n\to\infty} \frac{1}{2+\cos^2(n)} \neq 0$ (does not exist, in fact), so series diverges.

**c**. [4 points] For which values of $a$ does the series

$$\sum_{n=1}^{\infty} \frac{a^n}{3^n} = \frac{a}{3} + \frac{a^2}{9} + \frac{a^3}{27} + \cdots$$

converge? For the values of $a$ where the series converges, find the sum of the series.

*Solution:* Geometric series, converges when $|r| = \left|\frac{a}{3}\right| < 1$, so converges on the interval $-3 < a < 3$. Converges to

$$\frac{a}{3}\left(\frac{1}{1-\frac{a}{3}}\right) = \frac{a}{3}\left(\frac{3}{3-a}\right) = \frac{a}{3-a}.$$