7. [14 points] You want to open a savings account to deposit 1000 dollars. Three banks offer the following options:

a. [3 points] Bank A offers its clients a savings account that earns 1.5% per year compounded annually. Define the sequence A_n to be the amount of money in the savings account n years after you deposit your 1000 dollars. Find a formula for A_n.

Solution: $A_n = 1000(1.015)^n$

b. [7 points] Bank B offers its clients a savings account that earns 2% per year compounded annually. At the end of each year, after the bank deposits the interest you earned, it withdraws a 1 dollar service fee from the account. Define the sequence B_n to be the amount of money, right after the service fee deduction, in the savings account n years after you deposit your 1000 dollars. Find B_1, B_2, B_3 and a **closed form** formula for B_n.

Solution:

$$B_1 = 1000(1.02) - 1 = 1019.$$

$$B_2 = (1000(1.02) - 1)(1.02) - 1 = 1000(1.02)^2 - (1 + 1.02) = 1038.38.$$

$$B_3 = \left(1000(1.02)^2 - (1 + 1.02)\right)(1.02) - 1 = 1000(1.02)^3 - (1 + 1.02 + 1.02^2)$$
$$= 1058.15.$$

$$\vdots$$

$$B_n = 1000(1.02)^n - (1 + 1.02 + 1.02^2 + \cdots + 1.02^{n-1}) = 1000(1.02)^n - \frac{1 - 1.02^n}{1 - 1.02}$$

c. [4 points] Bank C offers its clients a savings account that earns interest continuously at a rate of 1.5% of the current balance per year. At the same time, the bank withdraws a service fee from the account at a rate of 1 dollar per year continuously. Let $M(t)$ be the amount of money in the savings account t years after you deposit your 1000 dollars. Write the differential equation satisfied by $M(t)$. Include initial conditions.

Solution: $\dfrac{dM}{dt} = 0.015M - 1, \qquad M(0) = 1000.$