

8. [10 points] A solid S is obtained by rotating the region bounded by the curve $y = 1 - x^2$, the line $x = 0$, and the line $y = 0$ around the y -axis. The density of the solid is given by $\delta(y) = 1 + y$.

- a. [5 points] Write a definite integral that gives the mass of the solid S .

Solution:

$$\int_0^1 \pi(1-y)(1+y)dy$$

- b. [5 points] Find formulas for \bar{x} and \bar{y} , the x and y coordinates of the center of mass of the solid S . The formulas may be written in terms of definite integrals, which you do not need to evaluate.

Solution: $\bar{x} = 0$ by symmetry.

$$\bar{y} = \frac{\int_0^1 \pi y(1-y)(1+y)dy}{\int_0^1 \pi(1-y)(1+y)dy}$$