8. [12 points]

a. [4 points] Let a be a positive constant. Determine the first three nonzero terms of the Taylor series for

$$f(x) = \frac{1}{(1+ax^2)^4}$$

centered at $x = 0$. Show all your work. Your answer may contain a.

b. [2 points] What is the radius of convergence of the Taylor series for $f(x)$? Your answer may contain a.

c. [3 points] Determine the first three nonzero terms of the Taylor series for

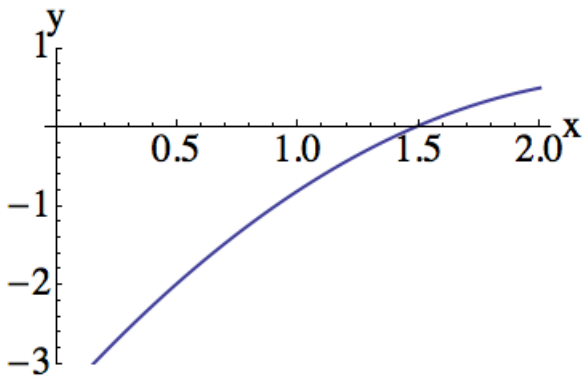
$$g(t) = \int_0^t \frac{1}{(1+ax^2)^4} dx,$$

centered at $t = 0$. Show all your work. Your answer may contain a.

d. [3 points] The degree-2 Taylor polynomial of the function $h(x)$, centered at $x = 1$, is

$$P_2(x) = a + b(x-1) + c(x-1)^2.$$

The following is a graph of $h(x)$:



What can you say about the values of a, b, c? You may assume a, b, c are nonzero. Circle your answers. No justification is needed.

a is: Positive Negative Not enough information

b is: Positive Negative Not enough information

c is: Positive Negative Not enough information