1. [8 points] Consider the differential equation and initial condition

$$\frac{dy}{dt} = At - y, \qquad y(0) = 5,$$

where $A > 0$ is a constant. This differential equation is not separable, but it is still possible to solve it using the following steps.

a. [5 points] Let $w(t) = \frac{dy}{dt}$. If you differentiate both sides of the differential equation above with respect to t, you obtain that the function $w(t)$ satisfies

$$\frac{dw}{dt} = A - w.$$

Find a general formula for $w(t)$, showing all work. Your answer may include A.

Solution:

$$\begin{aligned} \frac{dw}{dt} &= A - w \\ \frac{dw}{A-w} &= dt \\ -\ln|A-w| &= t + C \\ w(t) &= A - Be^{-t} \end{aligned}$$

b. [1 point] Given that $\frac{dy}{dt} = At - y$ and $y(0) = 5$, what must be the value of $w(0)$? Your answer may include A.

Solution: $w(0) = \frac{dy}{dt}|_{t=0} = At - y(t)|_{t=0} = 0 - 5 = -5.$

c. [2 points] Use the definition of $w(t)$ to obtain a formula for $y(t)$. Your answer may include A.

Solution:

$$y(t) = \int A - Be^{-t} dx = At + Be^{-t} + C$$

or using the differential equation

$$y(t) = At - \frac{dy}{dt} = At - (A - Be^{-t}) = A(t-1) + Be^{-t}.$$