10. [13 points] The blockbuster action movie *Mildred’s Adventures with Calculus!* was just released. During the first week after the premiere, 2.5 million people went to see it. The studio has conducted a study to gauge the impact of the film on audiences, and found that: *the number of tickets sold in a given week is 60% of the number of tickets sold the previous week.* Assume that this process repeats every week.

a. [5 points] Let \( p_k \) be the number of movie tickets, in millions, sold during the \( k \)th week after the premiere of the movie. Determine \( p_2 \), \( p_3 \) and a formula for \( p_k \).

**Solution:**

\[
\begin{align*}
p_1 &= 2.5 \\
p_2 &= 2.5 \times 0.6 = 2.5(0.6) \\
p_3 &= 2.5(0.6)^2 \\
p_k &= 2.5(0.6)^{k-1}.
\end{align*}
\]

b. [6 points] A movie ticket costs $8. Let \( T_n \) be the total amount of money earned in ticket sales, in millions of dollars, during the first \( n \) weeks the movie has been exhibited. Determine \( T_3 \) and a closed formula for \( T_n \). Show all your work.

**Solution:**

\[
\begin{align*}
T_1 &= 8(2.5) \\
T_2 &= 8(2.5 + 2.5 \times 0.6) \\
T_3 &= 8(2.5 + 2.5 \times 0.6 + 2.5 \times (0.6)^2) \\
T_n &= 8(2.5 + 2.5 \times 0.6 + 2.5 \times (0.6)^2 + \cdots + 2.5 \times (0.6)^{n-1}) \\
T_n &= 8(2.5) \frac{1 - (0.6)^n}{1 - 0.6} = 50(1 - (0.6)^n)
\end{align*}
\]

c. [2 points] Determine the value of \( \lim_{n \to \infty} T_n \).

**Solution:**

\[
\lim_{n \to \infty} T_n = \lim_{n \to \infty} 50(1 - (0.6)^n) = 50
\]