

3. [8 points] Indicate if each of the following is true or false by circling the correct answer. No justification is required.

- a. [2 points] The equation $y^3 - x^3 = xy$ in Cartesian coordinates can be written in polar coordinates as

$$r = \frac{\sin \theta \cos \theta}{\sin^3 \theta - \cos^3 \theta}.$$

True

False

Solution: Let $x = r \cos \theta$ and $y = r \sin \theta$, then $y^3 - x^3 = (r \sin \theta)^3 - (r \cos \theta)^3 = r^3(\sin^3 \theta - \cos^3 \theta)$, $xy = r^2 \sin \theta \cos \theta$, then $r = \frac{\sin \theta \cos \theta}{\sin^3 \theta - \cos^3 \theta}$.

- b. [2 points] If $g(x) = \int_1^x f(t) dt$, then $g(4) - g(2) = \int_2^4 f(t) dt$.

True

False

Solution: Since $g'(x) = f(x)$, then $g(x)$ is an antiderivative of $f(x)$. By the Fundamental Theorem of Calculus $\int_2^4 f(t) dt = g(4) - g(2)$.

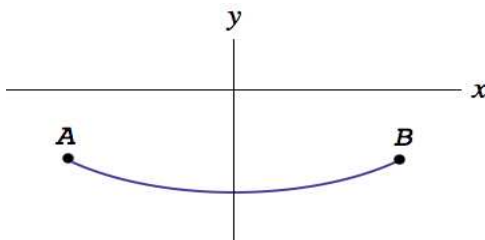
- c. [2 points] The function $h(x) = \int_0^{\sin x} e^{-t^2} dt$ has a local maximum at $x = \frac{\pi}{2}$.

True

False

Solution: Since $h'(x) = e^{-(\sin x)^2} \cos x$ and $h'(\frac{\pi}{2}) = 0$ and $h'(x)$ changes signs from positive to negative at $x = \frac{\pi}{2}$. Hence $h(x)$ has a local maximum at $x = \frac{\pi}{2}$.

- d. [2 points] The graph of the parametric equations $x = f(t)$ and $y = f'(t)$ for some function $f(t)$ is shown below:



As t increases, the curve is traced from A to B .

True

False

Solution: Since the graph is below the y -axis, then $y = f'(t) < 0$. Hence $f(t)$ is decreasing, and since $x = f(t)$, then as t increases, the values of x decrease. Hence the curve is traced from B to A .