4. [9 points]
Determine if each of the following sequences is increasing, decreasing or neither, and whether it converges or diverges. If the sequence converges, identify the limit. Circle all your answers. No justification is required.

a. [3 points] $a_n = \displaystyle\int_1^{n^3} \frac{1}{(x^2+1)^{\frac{1}{5}}}\,dx.$

Converges to ________________ **DIVERGES**.

INCREASING Decreasing Neither.

Solution: (Not required)
Since $\displaystyle\int_1^{\infty} \frac{1}{(x^2+1)^{\frac{1}{5}}}dx \approx \int_1^{\infty} \frac{1}{x^{\frac{2}{5}}}dx$ which diverges by p-test (with $p = \frac{2}{5} \leq 1$). Hence, $\displaystyle\lim_{n\to\infty} a_n = \lim_{n\to\infty} \int_1^{n^3} \frac{1}{(x^2+1)^{\frac{1}{5}}}dx = \int_1^{\infty} \frac{1}{(x^2+1)^{\frac{1}{5}}}dx = \infty$. The sequence is increasing since $\dfrac{1}{(x^2+1)^{\frac{1}{5}}} > 0$.

b. [3 points] $b_n = \displaystyle\sum_{k=0}^{n} \frac{(-1)^k}{(2k+1)!}.$

CONVERGES TO $\sin 1$ Diverges.

Increasing Decreasing **NEITHER**.

Solution: (Not required)
Since $\sin x = \displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$, then $\displaystyle\lim_{n\to\infty} b_n = \sum_{k=0}^{\infty} \frac{(-1)^k (1)^{2k+1}}{(2k+1)!} = \sin 1$. Since the series is alternating, the sequence is neither increasing or decreasing.

c. [3 points] $c_n = \cos(a^n)$, where $0 < a < 1$.

CONVERGES TO 1 Diverges.

INCREASING Decreasing Neither.

Solution: (Not required)
Since $\displaystyle\lim_{n\to\infty} a^n = 0$, then $\displaystyle\lim_{n\to\infty} \cos a^n = \cos 0 = 1$.