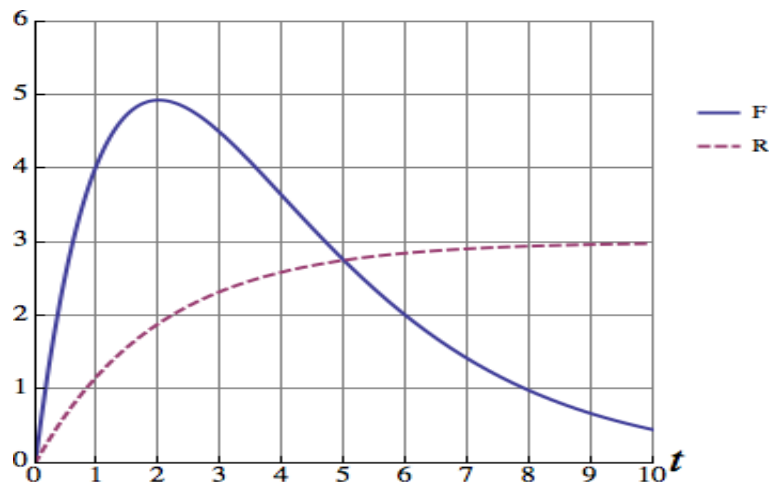


5. [12 points] The motor inside a submersible produces toxic fumes at a rate of $F(t)$ mg per minute, t minutes after the motor is activated. The submersible is equipped with an air filter system that removes the fumes at a rate $R(t)$ mg per minute. The total volume of air inside the submersible is 10 m^3 , and remains constant. The graph of $F(t)$ (solid line) and $R(t)$ (dashed line) are shown below.



- a. [4 points] Estimate the quantity of toxic fumes (in mg) produced by the motor during the first 4 minutes using the Midpoint rule with 2 subintervals. Write all the terms in your sum.

Solution: Toxic fumes $\approx 2(4 + 4.5) = 17$ mg

- b. [2 points] Is your estimate guaranteed to be an overestimate, underestimate or is it not possible to tell? Justify.

Solution: It is an overestimate since the $F(t)$ is concave down in $[0, 4]$.

- c. [2 points] At what time is the quantity of fumes in the submersible greatest?

Solution: $t = 5$ minutes.

- d. [4 points] Let $C(t)$ be the *concentration* of the fumes (in mg per m^3) in the air inside the submersible, t minutes after the motor is activated. Find a formula for $C(t)$.

Solution:

$$C(t) = \frac{1}{10} \int_0^t F(x) - R(x) dx.$$