5. [12 points] The motor inside a submersible produces toxic fumes at a rate of \( F(t) \) mg per minute, \( t \) minutes after the motor is activated. The submersible is equipped with an air filter system that removes the fumes at a rate \( R(t) \) mg per minute. The total volume of air inside the submersible is 10 m\(^3\), and remains constant. The graph of \( F(t) \) (solid line) and \( R(t) \) (dashed line) are shown below.

![Graph of F(t) and R(t)]

a. [4 points] Estimate the quantity of toxic fumes (in mg) produced by the motor during the first 4 minutes using the Midpoint rule with 2 subintervals. Write all the terms in your sum.

**Solution:** Toxic fumes \( \approx 2(4 + 4.5) = 17 \) mg

b. [2 points] Is your estimate guaranteed to be an overestimate, underestimate or is it not possible to tell? Justify.

**Solution:** It is an overestimate since the \( F(t) \) is concave down in \([0, 4]\).

c. [2 points] At what time is the quantity of fumes in the submersible greatest?

**Solution:** \( t = 5 \) minutes.

d. [4 points] Let \( C(t) \) be the concentration of the fumes (in mg per m\(^3\)) in the air inside the submersible, \( t \) minutes after the motor is activated. Find a formula for \( C(t) \).

**Solution:**

\[
C(t) = \frac{1}{10} \int_0^t F(x) - R(x) \, dx.
\]