7. [8 points] Consider the power series

$$\sum_{n=1}^{\infty} \frac{2^n}{3n}(x-5)^n.$$

In the following questions, support your answers by stating and properly justifying any test(s), facts and computations you use to prove convergence or divergence. Show all your work.

a. [4 points] Find the radius of convergence of the power series.

Solution:

$$\lim_{n\to\infty}\left|\frac{\frac{2^{n+1}}{3(n+1)}(x-5)^{n+1}}{\frac{2^n}{3n}(x-5)^n}\right| = |x-5|\lim_{n\to\infty}\frac{2n}{n+1} = 2|x-5| \qquad \text{then} \qquad \frac{1}{R}=2$$

or

$$R = \lim_{n\to\infty}\left|\frac{\frac{2^n}{3n}}{\frac{2^{n+1}}{3(n+1)}}\right| = \lim_{n\to\infty}\frac{n+1}{2n} = \frac{1}{2}.$$

Radius of convergence=0.5

b. [4 points] Find the interval of convergence of the power series. Make sure to cite all the tests you use to find your answer.

Solution: Testing the endpoints:

•$x = 4.5$: $\displaystyle\sum_{n=1}^{\infty}\frac{2^n}{3n}(4.5-5)^n = \frac{1}{3}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ converges by alternating series test.

•$x = 5.5$: $\displaystyle\sum_{n=1}^{\infty}\frac{2^n}{3n}(5.5-5)^n = \frac{1}{3}\sum_{n=1}^{\infty}\frac{1}{n}$ diverges by p-series test $p = 1 \leq 1$.

Interval of convergence: $4.5 \leq x < 5.5$.