8. [12 points]
   
   a. [4 points] Let $a$ be a positive constant. Determine the first three nonzero terms of the
   Taylor series for
   
   $$f(x) = \frac{1}{(1 + ax^2)^4}$$
   
   centered at $x = 0$. Show all your work. Your answer may contain $a$.
   
   Solution: Using the binomial series $(1 + u)^p$ with $u = ax^2$ and $p = -4$
   
   $$f(x) = \frac{1}{(1 + ax^2)^4} \approx 1 + pu + \frac{p(p-1)}{2} u^2 = 1 - 4ax^2 + 10a^2 x^4$$
   
   b. [2 points] What is the radius of convergence of the Taylor series for $f(x)$? Your answer
   may contain $a$.
   
   Solution: Since the interval of convergence of the binomial series is $-1 < u < 1$, then
   the interval of convergence of the series for $f(x)$ is $-1 < ax^2 < 1$, or $-\sqrt{\frac{1}{a}} < x < \sqrt{\frac{1}{a}}$.
   
   Hence the radius of convergence is $R = \sqrt{\frac{1}{a}}$.
   
   c. [3 points] Determine the first three nonzero terms of the Taylor series for
   
   $$g(t) = \int_0^t \frac{1}{(1 + ax^2)^4} dx,$$
   
   centered at $t = 0$. Show all your work. Your answer may contain $a$.
   
   Solution:
   
   $$g(t) = \int_0^t \frac{1}{(1 + ax^2)^4} dx \approx \int_0^t 1 - 4ax^2 + 10a^2 x^4 dx = t - \frac{4}{3} ax^3 + 2a^2 x^5 \bigg|_0^t = t - \frac{4}{3} at^3 + 2a^2 t^5$$
d. [3 points] The degree-2 Taylor polynomial of the function $h(x)$, centered at $x = 1$, is

$$P_2(x) = a + b(x - 1) + c(x - 1)^2.$$ 

The following is a graph of $h(x)$:

What can you say about the values of $a, b, c$? You may assume $a, b, c$ are nonzero. Circle your answers. No justification is needed.

Solution:

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<thead>
<tr>
<th></th>
<th>Positive</th>
<th>NEGATIVE</th>
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<tbody>
<tr>
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</tr>
<tr>
<td>$b$</td>
<td>Positive</td>
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</tr>
<tr>
<td>$c$</td>
<td>Positive</td>
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<td>Not enough information</td>
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