9. [13 points] Olive oil have been poured into the Math Department’s starfish aquarium! The shape of the aquarium is a solid of revolution, obtained by rotating the graph of $y = x^4$ for $0 \leq x \leq 1$ around the $y$-axis. Here $x$ and $y$ are measured in meters.

The aquarium contains water up to a level of $y = 0.6$ meters. There is a layer of oil of thickness 0.2 meters floating on top of the water. The water and olive oil have densities 1000 and 800 kg per m$^3$, respectively. Use the value of $g = 9.8$ m per s$^2$ for the acceleration due to gravity.

a. [6 points] Give an expression involving definite integrals that computes the total mass of the water in the aquarium.

$$\text{Solution: } \text{Mass}_{\text{water}} = \int_0^{0.6} \pi (\sqrt[4]{y})^2 (1000) \, dy = \int_0^{0.6} \pi \sqrt[4]{y} (1000) \, dy$$

b. [7 points] Give an expression involving definite integrals that computes the work necessary to pump all the olive oil to the top of the aquarium.

$$\text{Solution: } \text{Work}_{\text{oil}} = \int_{0.6}^{0.8} \pi (\sqrt[4]{y})^2 (800)(9.8)(1 - y) \, dy = \int_{0.6}^{0.8} \pi \sqrt[4]{y} (800)(9.8)(1 - y) \, dy$$