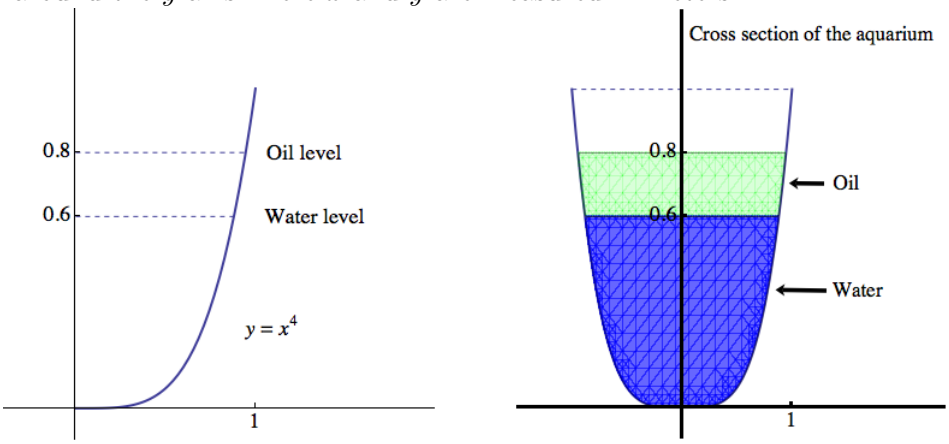
9. [13 points] Olive oil have been poured into the Math Department's starfish aquarium! The shape of the aquarium is a solid of revolution, obtained by rotating the graph of $y = x^4$ for $0 \le x \le 1$ around the y-axis. Here x and y are measured in meters.



The aquarium contains water up to a level of $y = 0.6$ meters. There is a layer of oil of thickness 0.2 meters floating on top of the water. The water and olive oil have densities 1000 and 800 kg per m^3, respectively. Use the value of $g = 9.8$ m per s^2 for the acceleration due to gravity.

a. [6 points] Give an expression involving definite integrals that computes the total mass of the water in the aquarium.

Solution: $\text{Mass}_{water} = \displaystyle\int_0^{0.6} \pi(\sqrt[4]{y})^2(1000)dy = \int_0^{0.6} \pi\sqrt{y}(1000)dy$

b. [7 points] Give an expression involving definite integrals that computes the work necessary to pump all the olive oil to the top of the aquarium.

Solution: $\text{Work}_{oil} = \displaystyle\int_{0.6}^{0.8} \pi(\sqrt[4]{y})^2(800)(9.8)(1-y)dy = \int_{0.6}^{0.8} \pi\sqrt{y}(800)(9.8)(1-y)dy$