11. [9 points] Circle all true statements.

a. [3 points] The integral $\int_0^1 \frac{1}{\sin(x)} \, dx$

   I. converges by the comparison test because $\frac{1}{\sin(x)} \leq C$ for some constant $C$ for $0 < x \leq 1$ and $\int_0^1 C \, dx$ converges.

   II. diverges by the comparison test because $\frac{1}{\sin(x)} \geq \frac{1}{x}$ for $0 < x \leq 1$ and $\int_0^1 \frac{1}{x} \, dx$ diverges.

   III. diverges because $\lim_{x \to 0} \frac{1}{\sin(x)} \neq 0$.

   IV. converges by the alternating series test because the values of $\sin(x)$ oscillate between $-1$ and $1$.

b. [3 points] The series $\sum_{n=0}^{\infty} \frac{e^{n^2}}{n!}$

   I. converges because $\lim_{n \to \infty} \frac{e^{n^2}}{n!} = 0$.

   II. converges because factorials grow faster than exponential functions.

   III. diverges by the ratio test.

   IV. diverges by the comparison test because $\frac{e^{n^2}}{n!} \geq e^n$ for $n = 0, 1, 2, 3, \ldots$ and $\sum_{n=0}^{\infty} e^n$ diverges.

c. [3 points] The differential equation $\frac{dy}{dt} = t(y - 2)(\ln(y))$ defined for $t > 0$ and $y > 0$ has

   I. an unstable equilibrium solution at $t = 0$.

   II. a stable equilibrium solution at $y = 2$.

   III. a stable equilibrium solution at $y = 1$.

   IV. an unstable equilibrium solution at $y = 2$. 