11. [9 points] Circle all true statements.

a. [3 points] The integral $\displaystyle\int_0^1 \frac{1}{\sin(x)} dx$

I. converges by the comparison test because $\dfrac{1}{\sin(x)} \leq C$ for some constant C for $0 < x \leq 1$ and $\displaystyle\int_0^1 C dx$ converges.

II. diverges by the comparison test because $\dfrac{1}{\sin(x)} \geq \dfrac{1}{x}$ for $0 < x \leq 1$ and $\displaystyle\int_0^1 \frac{1}{x} dx$ diverges.

III. diverges because $\displaystyle\lim_{x\to 0} \frac{1}{\sin(x)} \neq 0$.

IV. converges by the alternating series test because the values of $\sin(x)$ oscillate between -1 and 1.

b. [3 points] The series $\displaystyle\sum_{n=0}^{\infty} \frac{e^{n^2}}{n!}$

I. converges because $\displaystyle\lim_{n\to\infty} \frac{e^{n^2}}{n!} = 0$.

II. converges because factorials grow faster than exponential functions.

III. diverges by the ratio test.

IV. diverges by the comparison test because $\dfrac{e^{n^2}}{n!} \geq e^n$ for $n = 0, 1, 2, 3, ...$ and $\displaystyle\sum_{n=0}^{\infty} e^n$ diverges.

c. [3 points] The differential equation $\dfrac{dy}{dt} = t(y-2)(\ln(y))$ defined for $t > 0$ and $y > 0$ has

I. an unstable equilibrium solution at $t = 0$.

II. a stable equilibrium solution at $y = 2$.

III. a stable equilibrium solution at $y = 1$.

IV. an unstable equilibrium solution at $y = 2$.