

12. [8 points] Franklin, your friendly new neighbor, is building a large chicken sanctuary. You decide to help Franklin build a special chicken coop with volume (in cubic km) given by the integral

$$\int_0^1 x \sqrt{1 - \cos(x^2)} dx.$$

This integral is difficult to evaluate precisely, so you decide to use the methods you've learned this semester to help out Franklin. Your friend and president-elect, Kazilla, stops by to give you a hand. She suggests finding the 4th degree Taylor polynomial, $P_4(x)$, for the function $1 - \cos(x^2)$ near $x = 0$.

- a. [4 points] Find $P_4(x)$.

- b. [4 points] Substitute $P_4(x)$ for $1 - \cos(x^2)$ in the integral and compute the resulting integral by hand, showing all of your work.