

6. [8 points] Suppose that $f(x)$, $g(x)$, $h(x)$ and $k(x)$ are all positive, differentiable functions. Suppose that

$$0 < f(x) < \frac{1}{x} < g(x) < \frac{1}{x^2}$$

for all $0 < x < 1$, and that

$$0 < h(x) < \frac{1}{x^2} < k(x) < \frac{1}{x}$$

for $x > 1$. Determine whether the following statements are always, sometimes or never true by circling the appropriate answer. No justification is necessary.

- a. [2 points] $\int_0^1 g(x)dx$ converges.

Always

Sometimes

Never

- b. [2 points] $\int_0^1 f(x)dx$ diverges.

Always

Sometimes

Never

- c. [2 points] $\sum_{n=1}^{\infty} h(n)$ diverges.

Always

Sometimes

Never

- d. [2 points] $\sum_{n=1}^{\infty} k(n)$ converges.

Always

Sometimes

Never