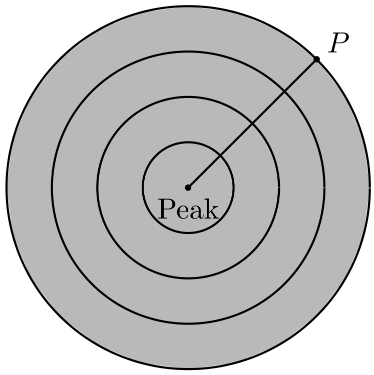
1. [11 points] You are standing on Lonely Hill while Franklin's robot army surrounds you. You have an Electro-Magnetic Pulse (EMP) device that deactivates all of the robots within a one mile radius of the peak of Lonely Hill as shown in the shaded region below. The point P is one mile from Lonely Hill for your reference. Let $R(x)$ give the density, in robots per square mile, of robots x miles from the peak of Lonely Hill. A table of values for $R(x)$ is given below.

x	0	0.25	0.5	0.75	1
$R(x)$	50	100	40	35	15



a. [3 points] Write an integral which gives the exact number of robots that are deactivated by the EMP. Your answer may contain the function $R(x)$. Do not evaluate your integral.

Solution:

$$2\pi \int_0^1 xR(x)dx$$

b. [5 points] Use the Trapezoidal method with as many subdivisions as possible to estimate the total number of robots that are deactivated by the EMP. Write out all of the terms for your estimate.

Solution:

$$LEFT(4) = 2\pi \cdot 0.25((0)50 + (0.25)100 + (0.5)40 + (0.75)35)$$
$$RIGHT(4) = 2\pi \cdot 0.25((0.25)100 + (0.5)40 + (0.75)35 + (1)15)$$
$$TRAP(4) = \frac{RIGHT(4) + LEFT(4)}{2} \approx 123.7$$

c. [3 points] You are very happy that the EMP worked. Let $H(t)$ give the level of happiness you feel t seconds after Franklin's robots are deactivated. Determine the value of a so that $H(t) = 3e^t + at^2 - 4t - 4$ is a solution to the differential equation

$$\frac{dH}{dt} = H + 2t^2.$$

Solution:

$$H'(t) = 3e^t + 2at - 4$$

$$\begin{aligned} 3e^t + 2at - 4 &= (3e^t + at^2 - 4t - 4) + 2t^2 \\ 2at &= at^2 - 4t + 2t^2 \\ 2a &= at - 4 + 2t \\ a(2-t) &= -2(2-t) \\ a &= -2 \end{aligned}$$

This implies that $at^2 + t^2 = 0$ and $2at = -4t$, so $a = -2$.