2. [6 points] Let \( f(x) = xe^{-x^2} \).

a. [4 points] Find the Taylor series of \( f(x) \) centered at \( x = 0 \). Be sure to include the first 3 nonzero terms and the general term.

**Solution:** We can use the Taylor series of \( e^y \) to find the Taylor series for \( e^{-x^2} \) by substituting \( y = -x^2 \).

\[
e^{-x^2} = \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!} = 1 + (-x^2) + \frac{(-x^2)^2}{2!} + \cdots + \frac{(-x^2)^n}{n!} + \cdots
\]

Therefore the Taylor series of \( xe^{-x^2} \) is

\[
x e^{-x^2} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!} = x - x^3 + \frac{x^5}{2!} + \cdots + \frac{(-1)^n x^{2n+1}}{n!} + \cdots
\]

b. [2 points] Find \( f^{(15)}(0) \).

**Solution:** We know that \( \frac{f^{(15)}(0)}{15!} \) will appear as the coefficient of the degree 15 term of the Taylor series. Using part (a), we see that the degree 15 term has coefficient \( \frac{-1}{15!} \).

Therefore

\[
f^{(15)}(0) = \frac{-15!}{15!} = -259, 459, 200
\]

3. [3 points] Determine the exact value of the infinite series

\[
1 - \frac{2}{1!} + \frac{4}{2!} - \frac{8}{3!} + \cdots + \frac{(-1)^n 2^n}{n!} + \cdots
\]

**Solution:** Notice that this is the Taylor series for \( e^y \) applied to \( y = -2 \). Therefore, the series has exact value \( e^{-2} \).