2. [6 points] Let $f(x) = xe^{-x^2}$.

a. [4 points] Find the Taylor series of $f(x)$ centered at $x = 0$. Be sure to include the first 3 nonzero terms and the general term.

Solution: We can use the Taylor series of e^y to find the Taylor series for e^{-x^2} by substituting $y = -x^2$.

$$e^{-x^2} = \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!} = 1 + (-x^2) + \frac{(-x^2)^2}{2!} + \cdots + \frac{(-x^2)^n}{n!} + \cdots$$

Therefore the Taylor series of xe^{-x^2} is

$$xe^{-x^2} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!} = x - x^3 + \frac{x^5}{2!} + \cdots + \frac{(-1)^n x^{2n+1}}{n!} + \cdots$$

b. [2 points] Find $f^{(15)}(0)$.

Solution: We know that $\frac{f^{(15)}(0)}{15!}$ will appear as the coefficient of the degree 15 term of the Taylor series. Using part (**a**), we see that the degree 15 term has coefficient $\frac{-1}{7!}$. Therefore

$$f^{(15)}(0) = \frac{-15!}{7!} = -259,459,200$$

3. [3 points] Determine the exact value of the infinite series

$$1 - \frac{2}{1!} + \frac{4}{2!} - \frac{8}{3!} + \cdots + \frac{(-1)^n 2^n}{n!} + \cdots$$

Solution: Notice that this is the Taylor series for e^y applied to $y = -2$. Therefore, the series has exact value e^{-2}.