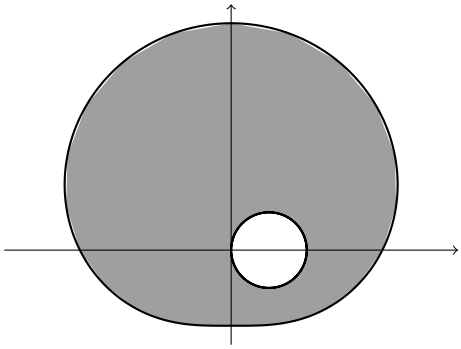
5. [11 points] Franklin's robot army is surrounding you!

a. [6 points] Consider the polar curves

$$r = \cos(\theta) \qquad\qquad r = \sin(\theta) + 2$$

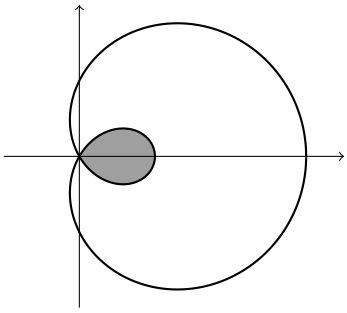
Franklin's robot army occupies the shaded region between these two curves. Write an expression involving integrals that gives the **area** occupied by Franklin's robot army. Do not evaluate any integrals.



Solution:

$$\text{Area} = \frac{1}{2}\int_0^{2\pi} (\sin(\theta) + 2)^2 d\theta - \frac{1}{2}\int_0^{\pi} (\cos(\theta))^2 d\theta$$

b. [5 points] Your friend, Kazilla, pours her magic potion on the ground. Suddenly, a flock of wild chickens surrounds you. The chickens occupy the shaded region enclosed within the polar curve $r = 1 + 2\cos(\theta)$ as shown below. Write an expression involving integrals that gives the **perimeter** of the region occupied by the flock of wild chickens. Do not evaluate any integrals.



Solution: We use the arc length formula:

$$\text{Arc Length} = \int_a^b \sqrt{(r(\theta))^2 + (r'(\theta))^2}\, d\theta$$

Note that $r'(\theta) = -2\sin(\theta)$. Also, the shaded region of lies between $\theta = 2\pi/3$ and $\theta = 4\pi/3$ (you can see this by setting $r(\theta) = 0$, and testing that $r(\pi) = -1$, so it lies on the boundary of the shaded region.)

$$\text{Arc Length} = \int_{2\pi/3}^{4\pi/3} \sqrt{(1 + 2\cos(\theta))^2 + (-2\sin(\theta))^2}\, d\theta$$