6. [8 points] Suppose that f(x), g(x), h(x) and k(x) are all positive, differentiable functions. Suppose that

$$0 < f(x) < \frac{1}{x} < g(x) < \frac{1}{x^2}$$

for all 0 < x < 1, and that

$$0 < h(x) < \frac{1}{x^2} < k(x) < \frac{1}{x}$$

for x > 1. Determine whether the following statements are always, sometimes or never true by circling the appropriate answer. No justification is necessary.

a. [2 points] $\int_0^1 g(x) dx$ converges.

Always

Sometimes

Never

b. [2 points]
$$\int_0^1 f(x) dx$$
 diverges.

Always

Sometimes

Never

c. [2 points]
$$\sum_{n=1}^{\infty} h(n)$$
 diverges.

Always

Sometimes

Never

d. [2 points]
$$\sum_{n=1}^{\infty} k(n)$$
 converges.

Always

Sometimes

Never