7. [14 points] Chickens continue to appear around you, and Franklin’s army is hesitant to advance.

a. [6 points] Let \( F(t) \) give the total number of chickens that have arrived after \( t \) seconds. You observe that \( F(t) \) obeys the following differential equation

\[
\frac{dF}{dt} = e^{-F} t^2.
\]

If there are initially 20 chickens, find a formula (in terms of \( t \)) for \( F(t) \).

**Solution:**

\[
\int e^F \, df = \int t^2 \, dt \\
e^F = \frac{t^3}{3} + C \\
F(t) = \ln\left(\frac{t^3}{3} + C\right)
\]

Since \( F(0) = 20 \), we see that

\[
20 = \ln(C)
\]

so \( C = e^{20} \), and

\[
F(t) = \ln\left(\frac{t^3}{3} + e^{20}\right)
\]

b. [4 points] A large, familiar-looking chicken steps forward from the flock and clucks, “Koo Koo Katcha!” This large chicken waddles towards Franklin following the parametric equations

\[
x(t) = \frac{\sin(\pi t) + 1}{\pi} \\
y(t) = \ln(t + 1)
\]

where \( t \) is the time, in seconds, after the chicken steps forward from the flock and both \( x \) and \( y \) are measured in feet. Find the chicken’s speed 10 seconds after it steps forward. Include units.

**Solution:**

\[
x'(t) = \cos(\pi t) \\
y'(t) = \frac{1}{t + 1}
\]

Now we plug these into the speed formula

\[
\text{Speed} = \sqrt{(x'(t))^2 + (y'(t))^2}
\]

when \( t = 10 \).

\[
\text{Speed} = \sqrt{(\cos(10\pi))^2 + \left(\frac{1}{11}\right)^2} = \frac{\sqrt{122}}{11}
\]
c. [4 points] Franklin says, “BEEP BOOP BEEP. YOU’RE RIGHT, WHAT HAVE I BECOME?” A single robot tear falls from Franklin’s robot eye. Consider the region in the $xy$-plane bounded by $y = \frac{\sin(x)}{x + 2}$, $x = \pi$, $x = 2\pi$, and the $x$-axis. The volume of Franklin’s tear is given by rotating this region around the $x$-axis. Write an integral giving the volume of Franklin’s tear. Do not evaluate this integral.

\[
\text{Solution:} \quad \int_{\pi}^{2\pi} \pi \left( \frac{\sin(x)}{x + 2} \right)^2 \, dx
\]