7. [14 points] Chickens continue to appear around you, and Franklin's army is hesitant to advance.

a. [6 points] Let $F(t)$ give the total number of chickens that have arrived after t seconds. You observe that $F(t)$ obeys the following differential equation

$$\frac{dF}{dt} = e^{-F}t^2.$$

If there are initially 20 chickens, find a formula (in terms of t) for $F(t)$.

Solution:

$$\int e^F df = \int t^2 dt$$
$$e^F = \frac{t^3}{3} + C$$
$$F(t) = \ln(\frac{t^3}{3} + C)$$

Since $F(0) = 20$, we see that

$$20 = \ln(C)$$

so $C = e^{20}$, and

$$F(t) = \ln(\frac{t^3}{3} + e^{20})$$

b. [4 points] A large, familiar-looking chicken steps forward from the flock and clucks, "Koo Koo Katcha!". This large chicken waddles towards Franklin following the parametric equations

$$x(t) = \frac{\sin(\pi t) + 1}{\pi} \qquad y(t) = \ln(t+1)$$

where t is the time, in seconds, after the chicken steps forward from the flock and both x and y are measured in feet. Find the chicken's speed 10 seconds after it steps forward. Include units.

Solution:

$$x'(t) = \cos(\pi t) \qquad y'(t) = \frac{1}{t+1}$$

Now we plug these into the speed formula

$$\text{Speed} = \sqrt{(x'(t))^2 + (y'(t))^2)}$$

when $t = 10$.

$$\text{Speed} = \sqrt{(\cos(10\pi))^2 + (\frac{1}{11})^2} = \frac{\sqrt{122}}{11}$$

c. [4 points] Franklin says, "`BEEP BOOP BEEP. YOU'RE RIGHT, WHAT HAVE I BECOME?`" A single robot tear falls from Franklin's robot eye. Consider the region in the xy-plane bounded by $y = \dfrac{\sin(x)}{x+2}$, $x = \pi$, $x = 2\pi$, and the x-axis. The volume of Franklin's tear is given by rotating this region around the x-axis. Write an integral giving the volume of Franklin's tear. Do not evaluate this integral.

Solution:

$$\int_{\pi}^{2\pi} \pi \left(\frac{\sin(x)}{x+2}\right)^2 dx$$