8. [7 points] Consider the power series \( \sum_{n=1}^{\infty} \frac{(x + 2)^n}{3^n n} \).

a. [2 points] At which \( x \)-value is the interval of convergence of this power series centered? 

Solution: This power series is centered on \( x = -2 \).

b. [5 points] The radius of convergence for the power series \( \sum_{n=1}^{\infty} \frac{(x + 2)^n}{3^n n} \) is 3. Find the interval of convergence for this power series. Thoroughly justify your answer.

Solution: Since the radius of convergence for this power series is 3 and it is centered on \( x = 2 \), the interval of convergence contains the open interval \((-2 - 3, -2 + 3) = (-5, 1)\). Now we only need to check the endpoints \( x = -5 \) and \( x = 1 \).

- For \( x = 1 \): \( \sum_{n=1}^{\infty} \frac{(1 + 2)^n}{3^n n} = \sum_{n=1}^{\infty} \frac{1}{n} \) diverges by the \( p \)-test with \( p = 1 \) (this is the harmonic series).
- For \( x = -5 \): \( \sum_{n=1}^{\infty} \frac{(-5 + 2)^n}{3^n n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \) which converges by the alternating series test.

Therefore, the interval of convergence for this power series is \([-5, 1)\).

9. [5 points] Find the radius of convergence for the power series \( \sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2 x^{2n}} \).

Solution: Let the \( n \)-th term be denoted by \( a_n \)

\[
\frac{|a_{n+1}|}{|a_n|} = \frac{(2(n + 1))! x^{2(n+1)}}{(n + 1)!^2} \cdot \frac{(n!)^2}{(2n)! x^{2n}} = \frac{(2n + 2)(2n + 1)x^2}{((n + 1)^2)}
\]

Therefore, we can use the ratio test:

\[
\lim_{n \to \infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n \to \infty} \left| \frac{(2n + 2)(2n + 1)x^2}{((n + 1)^2)} \right| = 4x^2.
\]

So this series converges for \( x \) with \( 4x^2 < 1 \), or rather with \( x^2 < \frac{1}{4} \) which implies that the radius of convergence is \( 1/2 \).