

3. [13 points]

- a. [4 points] The number of people R that have heard a rumor increases at a rate proportional to the product of the number of people that have heard the rumor and the number of people that haven't yet heard the rumor. Write a differential equation for R which models the scenario described assuming that the total number of people is 1,000. Use $k > 0$ for the constant of proportionality.

$$\frac{dR}{dt} = \underline{\hspace{10cm}}$$

- b. [4 points] For what values of A, B is $y(t) = At \cos t + Bt$ a solution to the differential equation $ty' = y + t^2 \sin t$ satisfying the initial condition $y\left(\frac{\pi}{2}\right) = 2\pi$? Be sure to show your work.

$$A = \underline{\hspace{10cm}}$$

$$B = \underline{\hspace{10cm}}$$

- c. [5 points] Find the solution to the differential equation

$$e^{-x} + y^2 \frac{dy}{dx} = 0, \quad \text{with initial condition } y(0) = 2.$$

$$y = \underline{\hspace{10cm}}$$