1. [10 points] Paul "Stretch" Cassenick and Stephen "Dee" Boxer never did have a third boxing match. Stephen chose a life of pacifism at a monastery in a far off country, and Paul became the czar of a Calculus 2 course at a large public university.

a. [6 points] To stay awake on the day of the final exam for his Calculus 2 class, Paul uses an intravenous drip of caffeine. Some values for the rate at which Paul is being given caffeine, $r(t)$, in mg/hour, t hours after he wakes up on the day of the exam are given in the table below. Assume the function $r(t)$ is differentiable and that its concavity does not change.

t	0	2	4	6	8
$r(t)$	95	68	50	40	35

Approximate the amount of caffeine given to Paul during the first 8 hours after he wakes up on the day of the exam using MID(n) with as many subdivisions as possible given the data in the table. Write out all the terms in your sum. Indicate whether your approximation is an over- or underestimate by circling the correct answer.

overestimate **underestimate**

Solution: The maximum number of subdivisions for MID is 2, and

$$\begin{aligned} \text{MID}(2) &= (4)(r(2) + r(6)) \\ &= (4)(68 + 40) \\ &= 432 \text{ mg} \end{aligned}$$

Since $r'(t)$ is increasing, $r(t)$ must be concave up. Thus MID(2) gives an underestimate.

b. [4 points] As part of his way to enlightenment, Stephen is digging a hole in the ground which will be the foundation for the new temple at his monastery. Suppose the soil has density $\delta(x)$ kg/m^3 at a depth of x meters below ground level at the temple site. If Stephen is digging a circular hole with diameter 60 meters, write an expression involving integrals which represents the mass of soil he has dug through when the hole has depth h meters.

Solution: A thin slice of the dirt in the hole of thickness δx meters has a mass of $\pi(30)^2\delta(x)$ kg. So the total mass of soil removed from the hole at a depth of h meters is

$$\int_0^h \pi(30)^2\delta(x)dx \text{ kg}.$$