13. [10 points] Suppose  $a_n$  and  $b_n$  are sequences with the following properties.

• 
$$\sum_{n=1}^{\infty} a_n$$
 converges.

• 
$$n \le b_n \le e^n$$
.

For each of the following statements, decide whether the statement is always true, sometimes true, or never true. Circle your answer. No justification is necessary. **You only need to answer 5 of the 7 questions**. Only answer the 5 questions you want graded. If it is unclear which 5 questions are being answered, the first 5 questions you answer will be graded.

**a.** [2 points] The sequence  $\frac{1}{b_n}$  diverges.

**ALWAYS** 

**SOMETIMES** 

**NEVER** 

**b.** [2 points] The sequence  $a_n$  is bounded.

ALWAYS

**SOMETIMES** 

**NEVER** 

**c**. [2 points] The series  $\sum_{n=1}^{\infty} \frac{1}{b_n}$  diverges.

ALWAYS

SOMETIMES

**NEVER** 

**d.** [2 points] The series  $\sum_{n=1}^{\infty} e^{-a_n}$  converges.

**ALWAYS** 

SOMETIMES

**NEVER** 

**e.** [2 points] The series  $\sum_{n=1}^{\infty} a_n^2$  diverges.

ALWAYS

SOMETIMES

**NEVER** 

**f.** [2 points] The series  $\sum_{n=1}^{\infty} a_n b_n$  converges.

**ALWAYS** 

SOMETIMES

**NEVER** 

**g.** [2 points] The series  $\sum_{n=1}^{\infty} \frac{b_n}{n!}$  converges.

ALWAYS

**SOMETIMES** 

**NEVER**