2. [8 points] Let $f(x) = x^{2x}$. The first two derivatives of f are given below.

$$f'(x) = 2(1 + \ln x)x^{2x}$$
$$f''(x) = 2x^{2x-1} + 4(1 + \ln x)^2 x^{2x}$$

a. [4 points] Find the 2nd degree Taylor polynomial $P_2(x)$ of f centered at $x = 1$.

Solution: Using the formula for Taylor polynomials,

$$\begin{aligned} P_2(x) &= f(1) + \frac{f'(1)}{1!}(x-1) + \frac{f''(1)}{2!}(x-1)^2 \\ &= 1 + 2(x-1) + 3(x-1)^2 \end{aligned}$$

$P_2(x) =$ $1 + 2(x-1) + 3(x-1)^2$

b. [4 points] Find

$$\lim_{x\to 1} \frac{x^{2x} - 1}{3x - 3}.$$

Clearly show your reasoning. Your answer from part **(a)** may be helpful.

Solution:

$$\begin{aligned} \lim_{x\to 1} \frac{x^{2x} - 1}{3x - 3} &= \lim_{x\to 1} \frac{1 + 2(x-1) + 3(x-1)^2 - 1}{3(x-1)} \\ &= \lim_{x\to 1} \frac{2 + 3(x-1)}{3} \\ &= \frac{2}{3} \end{aligned}$$