4. [8 points] Let $f(x) = \sqrt[3]{1+2x^2}$.

a. [5 points] Find the first 3 nonzero terms of the Taylor series for f centered at $x = 0$.

Solution: Using the Taylor series for $(1+y)^{1/3}$ centered at $y = 0$,

$$\sqrt[3]{1+y} = 1 + \frac{1}{3}y + \frac{\left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)}{2!}y^2 + \dots$$
$$= 1 + \frac{y}{3} - \frac{y^2}{9} + \dots$$

Substituting $y = 2x^2$,

$$\sqrt[3]{1+2x^2} = 1 + \frac{2x^2}{3} - \frac{4x^4}{9} + \dots$$

b. [3 points] For what values of x does the Taylor series converge?

Solution: The binomial series for $\sqrt[3]{1+y}$ converges when $-1 < y < 1$. Substituting $y = 2x^2$, this converges when $1 < 2x^2 < 1$, or $\frac{-1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$.

5. [3 points] Determine the **exact** value of the infinite series

$$-1 + \frac{1}{3!} - \frac{1}{5!} + \dots + \frac{(-1)^{n+1}}{(2n+1)!} + \dots$$

No decimal approximations are allowed. You **do not** need to show your work.

Solution:

$$-1 + \frac{1}{3!} - \frac{1}{5!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n(-1)^{2n+1}}{(2n+1)!} = \sin(-1).$$