6. [4 points] For each of the following questions, circle the answer which correctly completes the statement. You **do not** need to show your work.

a. [2 points] The integral $\displaystyle\int_1^\infty \frac{\ln x}{x^{3/2}}dx$

[✓ **converges**] **diverges**

Solution: $\ln x \le x^{1/4}$ for sufficiently large values of x, so

$$\frac{\ln x}{x^{3/2}} \le \frac{x^{1/4}}{x^{3/2}} = \frac{1}{x^{5/4}}$$

eventually. Since $\int_1^\infty \frac{1}{x^{5/4}}\,dx$ converges by the p-test ($p = \frac{5}{4} > 1$), $\int_1^\infty \frac{\ln x}{x^{3/2}}\,dx$ also converges by direct comparison.

b. [2 points] The integral $\displaystyle\int_0^1 \frac{x}{x^2 + x^{3/2}}dx$

[✓ **converges**] **diverges**

Solution:

$$\frac{x}{x^2 + x^{3/2}} = \frac{1}{x + \sqrt{x}} \le \frac{1}{\sqrt{x}}$$

for all positive values of x. Since $\int_0^1 \frac{1}{\sqrt{x}}\,dx$ converges by the p-test ($p = \frac{1}{2} < 1$), $\int_0^1 \frac{x}{x^2+x^{3/2}}\,dx$ also converges by direct comparison.

7. [6 points] The power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{5^n n}$$

has a radius of convergence of 5. For each of the endpoints of the interval of convergence, fill in the first two blanks with the endpoint and the series at that endpoint (in sigma notation or by writing out the first 4 terms), and then indicate whether the series converges at that endpoint in the final blank. You **do not** need to show your work.

At the endpoint $x =$ **-4**, the series is $\sum_{n=1}^{\infty} \frac{1}{n}$

and that series **diverges**.

At the endpoint $x =$ **6**, the series is $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

and that series **converges**.