9. [4 points] Maria wants to get a tattoo of her favorite geometric shape. The shape happens to be the region enclosed by the rose $r = 2\cos(3\theta)$. In order to make sure that the tattoo turns out perfectly, the artist needs to know how much ink is necessary. Find an expression involving integrals that gives the total area of the shaded region depicted below.

Solution: Since $r = 2\cos(3\theta)$ completes one full cycle in π radians, the total area is $\frac{1}{2}\int_0^\pi (2\cos(3\theta))^2\, d\theta$.

$r = 2\cos(3\theta)$
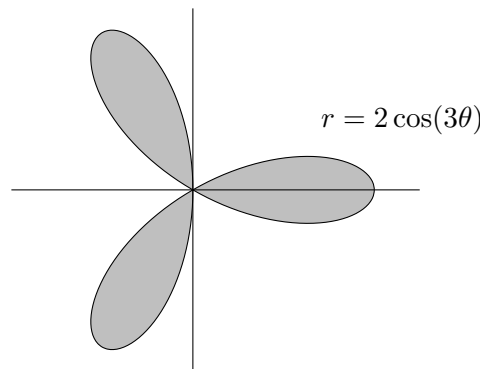
10. [4 points] The series

$$\sum_{n=0}^{\infty} \frac{9^n}{8^n + 10^n}$$

converges.

Use an appropriate series test to show that the series converges. Be sure to indicate which test(s) you are using. Also verify all hypotheses needed for the test, and justify the convergence/divergence of any other series you use.

Solution: $\frac{9^n}{8^n+10^n} \leq \frac{9^n}{10^n} = \left(\frac{9}{10}\right)^n$ for all values of n. Since $\sum_{n=0}^{\infty} \left(\frac{9}{10}\right)^n$ is a geometric series

with $|r| = \frac{9}{10} < 1$, it converges. Thus $\sum_{n=0}^{\infty} \frac{9^n}{8^n + 10^n}$ also converges by the Direct Comparison Test.