

7. [7 points] The *Legendre equation* is a differential equation that arises in the quantum mechanical study of the hydrogen atom. In one of its forms, the Legendre equation is

$$(1 - x^2)y'' - 2xy' + 12y = 0.$$

For this problem, let  $y$  be a solution to the Legendre equation satisfying  $y(\frac{1}{2}) = 2$  and  $y'(\frac{1}{2}) = 3$ . Assume that the Taylor series for  $y(x)$  about  $x = \frac{1}{2}$  converges to  $y(x)$  for all  $-\frac{1}{2} < x < \frac{3}{2}$ .

- a. [4 points] In the blank below, write down  $P_2(x)$ , the degree 2 Taylor polynomial of  $y(x)$  near  $x = \frac{1}{2}$ . Your answer should not contain the function  $y(x)$  or any of its derivatives.

$$P_2(x) = \underline{\hspace{10cm}}$$

- b. [3 points] Compute the limit

$$\lim_{x \rightarrow 1/2} \frac{y(x) - \frac{1}{2} - 3x}{(x - \frac{1}{2})^2}.$$