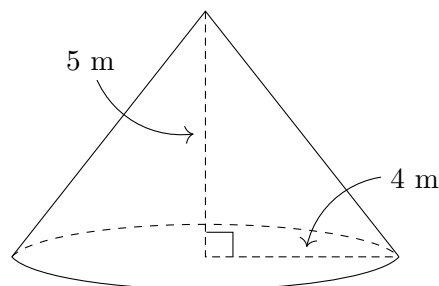


10. [14 points] The conical tank pictured below has a base of radius 4 meters and a height of 5 meters. It is filled to the top with a toxic liquid, PGM, which has a constant density of 1000kg/m^3 . The gravitational constant is $g = 9.8 \text{ m/s}^2$.



- a. [4 points] Write an expression for the volume of a circular slice of thickness Δh , a distance h meters from the base.

Solution: The volume is $\pi(4 - \frac{4}{5}h)^2 \Delta h \text{ m}^3$.

- b. [3 points] Dr. Durant is trying to take over Shamcorp by making a device using PGM. Dr. Durant must pump all the PGM to the top of the cone to complete his device. Write an expression involving integrals which gives the work Dr. Durant does pumping all of the liquid to the top of the container.

Solution: The work done is

$$9.8\pi \int_0^5 \left(4 - \frac{4}{5}h\right)^2 (5 - h) dh \text{ kJ}.$$

- c. [3 points] Write an expression **involving integrals** for A , the average radius of a circular slice of the cone for $0 \leq h \leq 5$.

Solution: We have

$$A = \frac{1}{5} \int_0^5 \left(4 - \frac{4}{5}h\right) dh \text{ m}.$$

- d. [4 points] Raymond Green is also building a PGM device, but he has a cylindrical container, oriented so that its circular base is on the ground, filled to the top with PGM. The container has a height of 5 meters and has radius A (the average radius of Dr. Durant's container). Whoever does the least amount of work pumping the PGM to the top of their container will rule Shamcorp. Will it be Dr. Durant or Raymond Green? Give a brief justification for your answer.

Solution: The volume of Durant's container is greater than that of Green's container; moreover, the center of mass of Durant's container is closer to the ground than that of Green's container. This means that Durant will do more work, so Green will rule ShamCorp.