1. [4 points] Suppose that the power series $\sum_{n=0}^{\infty} c_n(x-3)^n$ converges at $x = 6$ and diverges at $x = -2$. What can you say about the behavior of the power series at the following values of x? For each part, circle the correct answer. Ambiguous responses will be marked incorrect.

a. [1 point] At $x = -3$, the power series...

CONVERGES **[DIVERGES]** **CANNOT DETERMINE**

b. [1 point] At $x = 0$, the power series...

CONVERGES **DIVERGES** **[CANNOT DETERMINE]**

c. [1 point] At $x = 8$, the power series...

CONVERGES **DIVERGES** **[CANNOT DETERMINE]**

d. [1 point] At $x = 2$, the power series...

[CONVERGES] **DIVERGES** **CANNOT DETERMINE**

2. [5 points] Determine the **radius** of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2} x^{2n}.$$

Justify your work carefully and write your final answer in the space provided. Limit syntax will be enforced.

Solution: For $n = 0, 1, \ldots$, let $a_n = \dfrac{(2n)!}{(n!)^2}$. We have

$$\frac{|a_{n+1}|}{|a_n|} = \frac{(2(n+1))!}{((n+1)!)^2} \cdot \frac{(n!)^2}{(2n)!} = \frac{(2n+1)(2n+2)}{(n+1)^2} \to 4$$

as $n \to \infty$. Hence the radius of convergence is $\sqrt{\dfrac{1}{4}} = \dfrac{1}{2}$.

Radius of convergence = $\dfrac{1}{2}$