3. [8 points] For $n = 1, 2, 3, \ldots$ consider the sequence a_n given by

$$a_n = \frac{-1}{2^{(n+1)/2}} \text{ if } n \text{ is odd,} \qquad a_n = \frac{1}{3^{n/2}} \text{ if } n \text{ is even.}$$

a. [2 points] Write out the first 5 terms of the sequence a_n.

Solution: The first five terms are

$$-\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{8}.$$

b. [2 points] The series $\sum_{n=1}^{\infty} a_n$ is alternating. In a sentence or two, explain why the Alternating Series Test **cannot** be used to determine whether $\sum_{n=1}^{\infty} a_n$ converges or diverges.

Solution: The condition $|a_{n+1}| < |a_n|$ does not hold for all n. (It does not even hold eventually.)

c. [4 points] The series $\sum_{n=1}^{\infty} a_n$ converges. Show that it converges, either by using theorems about series, or by computing its exact value.

Solution: One possible answer is that the series is equal to the difference of two convergent geometric series:

$$\sum_{k=1}^{\infty} \frac{1}{3^k} - \sum_{k=1}^{\infty} \frac{1}{2^k} = \frac{\frac{1}{3}}{1-\frac{1}{3}} - \frac{\frac{1}{2}}{1-\frac{1}{2}} = -\frac{1}{2}.$$

Another answer uses the Comparison Test; for $n = 1, 2, \ldots$, let $b_n = \frac{1}{n^2}$, and notice that $|a_n| \leq b_n$ eventually. Since $\sum_{n=1}^{\infty} b_n$ converges by the p-Test ($p = 2$), $\sum_{n=1}^{\infty} |a_n|$ converges by comparison. Hence the original series converges.