4. [5 points] The following series diverges:

\[
\sum_{n=2}^{\infty} \frac{n}{n^2 + \ln(n)}.
\]

Use theorems about infinite series to show that the series diverges. Give full justification, showing all your work and indicating any theorems or tests that you use.

\textit{Solution:} One solution uses the Comparison Test. Notice that

\[
\frac{n}{n^2 + \ln(n)} \geq \frac{n}{n^2 + n^2} = \frac{1}{2n}
\]

for all \( n \geq 2 \). Since \( \sum_{n=1}^{\infty} \frac{1}{2n} \) diverges by the \( p \)-Test (\( p = 1 \)), the original series diverges by comparison.

Alternatively, let \( a_n = \frac{n}{n^2 + \ln(n)} \) and \( b_n = \frac{1}{n} \) for all \( n \geq 2 \), and notice that

\[
\lim_{n \to \infty} \frac{a_n}{b_n} = 1.
\]

Since \( \sum_{n=2}^{\infty} b_n \) converges by the \( p \)-Test (\( p = 1 \)), the original series diverges by the Limit Comparison Test.

5. [5 points] Let \( \alpha > 0 \) be a constant. Compute the first 3 terms of the Taylor series of \( f(x) = \frac{x}{\sqrt{1 + \alpha x}} \) about \( x = 0 \). Write the appropriate coefficients in the spaces provided.

\[
0 + 1 \cdot x + \frac{-\alpha}{2} \cdot x^2 + \cdots
\]