4. [5 points] The following series **diverges**:

$$\sum_{n=2}^{\infty} \frac{n}{n^2 + \ln(n)}.$$

Use theorems about infinite series to **show** that the series diverges. Give full justification, showing all your work and indicating any theorems or tests that you use.

Solution: One solution uses the Comparison Test. Notice that

$$\frac{n}{n^2 + \ln(n)} \geq \frac{n}{n^2 + n^2} = \frac{1}{2n}$$

for all $n \geq 2$. Since $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n}$ diverges by the p-Test ($p = 1$), the original series diverges by comparison.

Alternatively, let $a_n = \dfrac{n}{n^2 + \ln(n)}$ and $b_n = \dfrac{1}{n}$ for all $n \geq 2$, and notice that

$$\lim_{n \to \infty} \frac{a_n}{b_n} = 1.$$

Since $\displaystyle\sum_{n=2}^{\infty} b_n$ converges by the p-Test ($p = 1$), the original series diverges by the Limit Comparison Test.

5. [5 points] Let $\alpha > 0$ be a constant. Compute the first 3 terms of the Taylor series of $f(x) = \dfrac{x}{\sqrt{1 + \alpha x}}$ about $x = 0$. Write the appropriate coefficients in the spaces provided.

$$\underline{\quad 0 \quad} + \underline{\quad 1 \quad}\, x + \underline{\quad -\frac{\alpha}{2} \quad}\, x^2 + \cdots$$