7. [7 points] The *Legendre equation* is a differential equation that arises in the quantum mechanical study of the hydrogen atom. In one of its forms, the Legendre equation is

$$(1-x^2)y'' - 2xy' + 12y = 0.$$

For this problem, let y be a solution to the Legendre equation satisfying $y(\frac{1}{2}) = 2$ and $y'(\frac{1}{2}) = 3$. Assume that the Taylor series for $y(x)$ about $x = \frac{1}{2}$ converges to $y(x)$ for all $-\frac{1}{2} < x < \frac{3}{2}$.

a. [4 points] In the blank below, write down $P_2(x)$, the degree 2 Taylor polynomial of $y(x)$ near $x = \frac{1}{2}$. Your answer should not contain the function $y(x)$ or any of its derivatives.

$$P_2(x) = \underline{\quad 2 + 3\left(x - \frac{1}{2}\right) - 14\left(x - \frac{1}{2}\right)^2 \quad}$$

b. [3 points] Compute the limit

$$\lim_{x \to 1/2} \frac{y(x) - \frac{1}{2} - 3x}{(x - \frac{1}{2})^2}.$$

Solution: The limit is -14.